Wavelength-shift interferometry for distance measurements using the Fourier transform technique for fringe analysis

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The Fourier transform technique, originally developed for spatial fringe pattern analysis, has been applied to the analysis of a temporal fringe signal obtained by a wavelength-shift interferometer used for absolute distance measurements. It has been shown that the error caused by the nonlinear and time-varying current-wavelength characteristic of the laser diode can be removed by combining the Fourier transform technique with the reference technique. A novel technique for distance measurement based on multiple-beam interferometry has been proposed, and an experimental demonstration is given for a three-beam interferometer that includes a reference reflector as an integral part of the system. Error sources and the limitation of the technique are discussed.

Key words: Interferometry, distance measurement, fringe analysis, Fourier transform, interferometer.

I. Introduction

Recently several techniques of wavelength-shift interferometry have been proposed for absolute distance measurements. The distance measurable by these techniques ranges from a few millimeters to a few meters, which has been a rather critical range of measurement for most of the existing techniques. For example, it is too short for the sinusoidal light-intensity modulation technique to be useful, and yet it is too large to use conventional interferometric techniques whose sensitivity is as high as the wavelength of light. In such wavelength-shift interferometry, frequent use has been made of a laser diode as a frequency-tunable light source because the laser diode has the great advantage that the wavelength can be varied easily by changing the amount of the injection current. However, several problems exist associated with the use of a laser diode as a frequency-tunable light source for wavelength-shift interferometry. First, the relationship between the increment of the injection current and the wavelength shift caused by it is not necessarily such an ideal linear function as is often assumed. In addition, this current-wavelength characteristic may vary at each moment of the measurement unless the temperature of the laser diode is carefully stabilized. Second, the maximum usable range of the continuous wavelength shift is limited by mode hops to ~50-70 GHz. In some systems this restricts the resolution and/or the minimum distance measurable by the techniques. Another kind of problem we encounter in the interferometric measurement of distances much larger than the wavelength is that we have to reduce the sensitivity of the interferometer by introducing a synthetic wavelength, \( \Lambda = \lambda^2 / \Delta \lambda \), where \( \Delta \lambda \) is the amount of the wavelength shift given to the laser diode. This synthetic wavelength is effectively much larger than the actual wavelength \( \lambda \) of the laser diode. The distance is computed from the optical path difference \( L \) between the beam reflected from a target and the beam from a nearby reference and is determined by measuring the phase difference \( \Delta \phi \), which is given by

\[
\Delta \phi = 2\pi L / \lambda = 2\pi L \Delta \lambda / \lambda^2.
\] (1)

Therefore, the more the sensitivity is reduced, the more accurately the phase of the interferometric signal needs to be determined to retain the distance resolution. To meet this requirement, Kikuta et al. have proposed a synchronous detection scheme for the wavelength-shift interferometry and successfully...
measured distances over the range as mentioned earlier. Another technique based on coherence modulation of a laser diode source has been demonstrated by Hotate and Kamatani, which may be useful for the measurement of distances of spatially distributed scatterers. However, both of these techniques require acoustic optical modulators to give an additional frequency shift for the synchronous detection and/or for bandpass filtering of the interferometric signal; this makes the system rather complex. Other techniques have also been proposed that do not use such acoustic optical modulators. With these techniques, a large amount of current ramp is applied to the laser diode to produce a large wavelength shift Δλ, so that the phase φ in Eq. (1) varies much more than 2π, and the distance L is determined either by counting the number of fringes or by computing the position of a peak appearing in the frequency spectrum of the interferometric signal. The latter technique is sometimes referred to as optical frequency domain reflectometry (OFDR). Despite the ease of hardware implementation, the fringe counting technique cannot be used when the interferometric signal contains multiple-frequency components that are generated by interference between a reference beam and signal beams returning from multiple reflectors located at different distances. The interference between the multiple-signal beams themselves may also give rise to various frequency components. When the interferometric signal has such multiple-frequency components, the shape of the signal generally becomes complex and quite different from that of a simple sinusoid so that we cannot tell which part of the waveform corresponds to the fringe to be counted. Although the OFDR has the marked advantage that it allows us to read the distances of multiple reflectors from the locations of the spectrum peaks, the range resolution is limited by the spread of the spectrum peaks. This spread of the spectrum is unavoidable, because the mode hops do not allow us to give a large wavelength shift to the laser diode and also because the injection current ramp not only varies the wavelength but also modulates the light intensity.

Our purpose is to solve these problems by applying the Fourier transform technique (FTT) of fringe analysis to wavelength-shift interferometry. First, we review briefly the principles of wavelength-shift interferometry and the FTT. Next, we explain how these two techniques can be combined to solve the problems associated with the use of a laser diode. Finally, we describe experiments, including three-beam interferometry, that demonstrate the validity of the principle.

II. Wavelength-Shift Interferometry

We briefly review the principle of the wavelength-shift interferometry and identify the problems to be solved by the use of the FTT.

Let us first consider the simplest case of a two-beam interferometer with an optical path difference L. As we have stated, when a wavelength shift Δλ is given to the laser diode whose initial wavelength is λ, the phase of the interference fringe varies by the amount shown by Eq. (1). If a linear current ramp is applied to the laser diode, and if we assume that the current-wavelength characteristic is linear, the wavelength shift becomes a linear function of time t, Δλ = at, so that the phase of the interference fringe is given by

$$\phi(t) = \frac{2\pi L}{\lambda} - \frac{2\pi a t \lambda}{\lambda^2} = \phi_0 + 2\pi f_s t,$$

where \(\phi_0 = \frac{2\pi L}{\lambda}\) and

$$f_s = -\frac{a L}{\lambda^2},$$

where \(a\) is a proportionality constant. From Eq. (3) we see that the linear wavelength shift produces a constant carrier frequency \(f_s\), which is proportional to the optical path difference L. Thus we can obtain L by determining the carrier frequency \(f_s\), for example, by counting the number of fringes or by computing the position of the spectrum peak. Implicit in this principle is that we have knowledge of the exact values of the parameters \(a\) and \(\lambda\). However, these values may be different for every laser diode and may vary critically with the temperature of the diode. A reference technique proposed by Kobayashi may provide a solution to the problem. The basic idea of the technique is to use the laser diode as a common light source for two independent interferometers, one of which has a known optical path difference \(L_R\) and provides a reference frequency

$$f_{R_0} = -\frac{a L_R}{\lambda^2}.$$ 

By taking the ratio of \(f_s\) to \(f_{R_0}\) we can remove the indeterminate parameters \(a\) and \(\lambda\), so that

$$L = \left| \frac{f_s}{f_{R_0}} \right| L_R.$$ 

Since \(L_R\) is known, L can be obtained by measuring \(f_s\) and \(f_{R_0}\). Kobayashi determined this ratio by counting the number of fringes obtained from the two independent interferometers. As will be seen in our experiment on the three-beam interferometer, the fringe-counting technique cannot be used when the fringe signal contains multiple-frequency components that are generated by beams reflected from more than one reflector. We show that this difficulty is removed by use of the FTT.

III. Fourier Transform Technique

Originally the FTT was proposed for the analysis of spatial fringe patterns having a spatial carrier frequency and has been applied mainly to profilometry and wave-front shape measurements that require subfringe sensitivity. Here we describe its principle for a temporal signal to make the analysis relevant to the present purpose.

Let us consider the general case where the current-
wavelength characteristic of the laser diode may not be a strict linear function and may vary at every measurement. This time-dependent nonlinearity can be incorporated into the theory by considering the constant $a_0$ in Eq. (2) as a function of time $t$, which is expressed by

$$a(t) = a_0 + \Delta a(t),$$

(6)

where $a_0$ is a constant representing the time rate of the linear frequency shift, and $\Delta a(t)$ represents a deviation from the linearity. Equation (2) can now be rewritten as

$$\phi(t) = \phi_0 + 2\pi f_s t + \theta(t),$$

(7)

where

$$\theta(t) = -2\pi \Delta a(t)t/\lambda^2,$$

(8)

and $f_s$ is now redefined as

$$f_s = -a_0 L/\lambda^2.$$  

(9)

For simplicity we first treat the case of two-beam interference. The interferometric signal for this case is given by

$$g(t) = a(t) + b(t) \cos(2\pi f_s t + \theta(t) + \phi_0),$$

(10)

where $a(t)$ is the intensity sum of the two beams, and $b(t)$ is the amplitude of the interference term, both of which vary with the laser power when a ramp current is applied to the diode laser. These two are unwanted variations, and we are interested in the phase $\phi(t)$ [defined by Eq. (7)] of the cosine term that has information about $L$. The first problem is how to determine the phase $\phi(t)$ accurately, excluding the influence from other unwanted variations $a(t)$ and $b(t)$. We rewrite Eq. (10) in the following form for convenience:

$$g(t) = a(t) + c(t) \exp(2\pi j f_s t) + c^*(t) \exp(-2\pi j f_s t),$$

(11)

with

$$c(t) = \frac{1}{2}b(t) \exp[j(\theta(t) + \phi_0)],$$

(12)

where $*$ denotes a complex conjugate. Next we compute the Fourier transform of Eq. (11), which gives

$$G(f) = \int_{-\infty}^{\infty} g(t) \exp(-2\pi j f t) dt = A(f) + C(f - f_s) + C^*(-f + f_s),$$

(13)

where the uppercase letters denote the Fourier spectra. If the optical path difference $L$ is significantly larger than the synthetic wavelength $\Lambda$, and if $|a_0| \gg |\Delta a(t)|$, the carrier frequency $f_s$ becomes much larger than the spread of the spectra caused by the variations of $a(t)$, $b(t)$, and $\theta(t)$, so that the three spectra are separated by the carrier frequency $f_s$ as shown in Fig. 1. The principle of the OFDR is to determine $f_s$ from the peak position of the spectrum $C(f - f_s)$ and compute $L$ from Eq. (9). The range resolution of the OFDR is limited by the spread of the spectrum caused by the amplitude modulation $b(t)$ and the phase modulation $\theta(t)$ arising from the nonlinear current-wavelength characteristic. Furthermore, it should be noted that the position of the spectrum peak does not necessarily represent the correct distance because the phase modulation $\theta(t)$ may introduce an asymmetric spread of the spectrum. For example, the position of the spectrum will be shifted when $\theta(t)$ has a characteristic like frequency chirping. These problems can be solved by the following procedure. First, we select only one spectra $C(f - f_s)$ by filtering. Note that the unwanted intensity variation $a(t)$ has been filtered out in this stage. Next, we compute the inverse Fourier transform of $C(f - f_s)$ and obtain the analytic signal

$$c(t) \exp(2\pi j f_s t) = \frac{1}{2}b(t) \exp[j(2\pi f_s t + \theta(t) + \phi_0)].$$

(14)

Then we calculate a complex logarithm of this analytic signal:

$$\log[c(t) \exp(2\pi j f_s t)] = \log[b(t)] + j[2\pi f_s t + \theta(t) + \phi_0].$$

(15)

Now we have the phase of the fringe signal in the imaginary part completely separated from the unwanted amplitude variation $b(t)$ in the real part. Since the imaginary part of Eq. (15) calculated by computer gives the principal value of the phase with modulo $2\pi$, the phase $\phi(t)$ is wrapped into the range $[-\pi, \pi]$, so that its distribution has discontinuities with $2\pi$ phase jumps. This wrapped phase is corrected by using a phase unwrapping algorithm.11 Note that by this technique we can determine the phase $\phi(t)$ with the resolution exceeding $2\pi$.

Next we explain how to apply the reference technique to the phase obtained by the FTT. Again here we use one extra interferometer, which provides a reference phase signal:

$$\phi_0(t) = \phi_{\phi 0} + 2\pi f_s t + \theta_0(t),$$

(16)

where $\phi_{\phi 0} = 2\pi L_s/\lambda$ and

$$\theta_0(t) = -2\pi L_s \Delta a(t) t/\lambda^2,$$

(17)
and \( f_R \) is now redefined by
\[
f_R = -a_L L / \lambda^2. \tag{18}
\]

To eliminate the unknown constant phases \( \phi_R \) and \( \phi_{R0} \), we differentiate the phases and obtain instantaneous angular frequencies:
\[
\omega_R(t) = \frac{d\phi_R(t)}{dt} = 2\pi f_R + \frac{d\theta_R(t)}{dt} = -\frac{2\pi L \alpha(t)}{\lambda^2} \left[ a_R + \frac{d\alpha(t)}{dt} \right], \tag{19}
\]
\[
\omega_d(t) = \frac{d\phi_d(t)}{dt} = 2\pi f_d + \frac{d\theta_d(t)}{dt} = \frac{2\pi L \alpha(t)}{\lambda^2} \left[ a_R + \frac{d\alpha(t)}{dt} \right]. \tag{20}
\]

From the ratio of \( \omega_R(t) \) to \( \omega_d(t) \), we can compute the optical path difference \( L \) by
\[
L = \left[ \frac{\omega_R(t)}{\omega_d(t)} \right] L_R. \tag{21}
\]

It should be noted that, although the temperature-sensitive nonlinear current-wavelength characteristic of the laser diode may cause \( \omega_R(t) \) and \( \omega_d(t) \) to vary with time, their ratio remains constant. This makes it possible to improve accuracy simply by averaging the ratio over the time spent on the measurement. In other words, we can make use of full sets of data to determine the optical path difference \( L \) rather than determining it from a single set of data obtained at a certain instant of time. Thus, by combining the FFT with the reference technique, we can not only remove the influence of the nonlinear current-wavelength characteristics and the temperature-dependent wavelength fluctuations but also adopt simple averaging operations to improve the accuracy.

\section*{IV. Multiple-Beam Interferometry with Multiple Reflectors}

So far we have restricted our discussion to the case of two-beam interferometry in which we have only one target reflector besides a reference reflector. Let us now consider the more general case of multiple-beam interferometry with multiple reflectors, as illustrated in Fig. 2, and show that the FFT has a great advantage over the fringe-counting technique in such multiple-beam environments. Suppose we have \( N \) reflectors among which one serves as a reference reflector. Now the interferometric signal in Eq. (10) becomes
\[
g(t) = \alpha(t) + \sum_{m<n} b_{mn}(t) \cos[2\pi f_{mn} t + \theta_{mn}(t) + \phi_{mn}], \tag{22}
\]

where \( \alpha(t) \) is the intensity sum of the \( N \) beams, and \( b_{mn}(t) \) is the amplitude of the fringe caused by the interference of a pair of beams reflected from mirrors \( M_m \) and \( M_n \). Furthermore, \( \phi_{mn} = 2\pi L_{mn} / \lambda \) and
\[
\theta_{mn}(t) = -2\pi L_{mn} \Delta \alpha(t) t / \lambda^2, \tag{23}
\]

and \( f_{mn} \) is now redefined as
\[
f_{mn} = -a_L L_{mn} / \lambda^2. \tag{24}
\]

where \( L_{mn} \) is the optical path difference between the beams returning from mirrors \( M_m \) and \( M_n \). We have assumed that the beams generated by multiple reflections between the mirrors are so weak that their fringes can be neglected. Since the interferometric signal given by Eq. (22) is not a simple sinusoid and has multiple-frequency components, the fringe-counting technique cannot be applied. Now we show how the FFT can be used for the analysis of such a complex signal with multiple-frequency components. Again we compute the Fourier transform of Eq. (22), which gives
\[
G(f) = A(f) + \sum_{m<n} \left[ C_{mn}(f - f_{mn}) + C_{mn}(-(f + f_{mn}) \right], \tag{25}
\]

where \( C_{mn} \) is the Fourier spectrum of
\[
c_{mn}(t) = \frac{1}{2} b_{mn}(t) \exp \left[ j(\theta_{mn}(t) + \phi_{mn}) \right]. \tag{26}
\]

If the optical path differences \( L_{mn} \) are such that the separations between the carrier frequencies \( f_{mn} \) are much larger than the spread of the spectra caused by the variations of \( \alpha(t) \), \( b_{mn}(t) \), and \( \theta_{mn}(t) \), as shown in Fig. 3, we can use the same filtering and phase detection algorithm as has been described for the case of two-beam interferometry and obtain
\[
\phi_{mn}(t) = 2\pi f_{mn} t + \theta_{mn}(t) + \phi_{mn}. \tag{27}
\]

Among \( \frac{1}{2} \alpha(N - 1) \) possible combinations of \( L_{mn} \), we choose one, say \( L_{123} \), as a reference and use the reference technique
\[
L_{mn} = \left[ \frac{\omega_{mn}(t)}{\omega_{123}(t)} \right] L_{123}. \tag{28}
\]

Among \( \frac{1}{2} \alpha(N - 1) \) possible combinations of \( L_{mn} \), we choose one, say \( L_{123} \), as a reference and use the reference technique
\[
L_{mn} = \left[ \frac{\omega_{mn}(t)}{\omega_{123}(t)} \right] L_{123}. \tag{28}
\]
where $\omega_{mn} = \frac{d\phi_{mn}}{dt}$, and we assume that $L_{12}$ is a known reference optical path length.

V. Experiments

Experiments have been conducted to demonstrate the validity of the principle. First, we describe the experiment based on two-beam interferometry using two independent interferometers and show how the FTT is combined with the reference technique. Then we demonstrate through the experiment of three-beam interferometry that the FTT can be applied to multiple-beam interferometry, where the interferometric signal includes multiple-frequency components caused by the beams from multiple reflectors.

A. Combination of the FTT with the Reference Technique

As shown in Fig. 4 we used a pair of two-beam interferometers sharing a common light source, where one provides an interferometric signal generated by the beam from the target, and the other serves as a reference signal generator. The system is essentially the same as that used by Kobayashi, but now fringe counters are excluded from the system, and their function is replaced by the algorithm of the FTT performed by the computer. The light source is a single-longitudinal-mode laser diode (Hitachi HL7801E) with wavelength $\lambda = 780$ nm and power $P = 5$ mW. The laser diode was temperature controlled by a thermoelectric module to avoid mode hops by keeping the temperature variation to within $0.1^\circ$C. The beam form of the laser was collimated by collimator CL, split into two beams by beam splitter BS1, and introduced into the two independent Michelson interferometers. The reference interferometer has two fixed arms with a known optical path difference of 100 cm, while the other object interferometer has a fixed arm and a variable arm with a movable corner cube mirror. The laser diode was driven by a ramp current superimposed on the bias current. The ramp current had a triangular waveform with a repetition frequency of $f_{LD} = 125$ Hz and an amplitude $\Delta i = 10$ mA, which introduced a maximum wavelength shift of 0.06 nm. The two fringe signals were detected separately, analog-to-digital converted, and stored in the memory of a personal computer (NEC PC-9801).

Figures 5(a) and 5(b) show part of the fringe signals obtained, respectively, from the object interferometer with an optical path difference $L_s = 50$ cm and from the reference interferometer with an optical path difference $L_R = 100$ cm. The signals were preprocessed by an analog circuitry to remove the dc component and the effect of the laser power variations and were recorded for $T = 4$ ms over the rising period of the triangular current waveform. These fringe signals were analyzed by the FTT. The Fourier transforms of the fringe signals were computed by using a fast Fourier transform algorithm with 1024 sample points. Shown in Figs. 6(a) and 6(b) are the positive frequency Fourier spectra computed, respectively, for fringes (a) and (b) in Fig. 5. After selecting only the positive carrier-frequency spectra by the bandpass filter as shown in Fig. 6, we computed
instantaneous angular frequencies defined by Eqs. (19) and (20). The instantaneous angular frequencies \( \omega_o(t) \) and \( \omega_R(t) \) are shown in Fig. 7(a); 100 data points were eliminated from both ends because the Hanning window used for the fast Fourier transform calculation forced the fringe amplitude to be nearly zero in these regions and made the computed phase data unreliable. Note that both \( \omega_o(t) \) and \( \omega_R(t) \) increase with fluctuations and are not constant. These variations are caused by the nonlinear and time-varying current-wavelength characteristic of the laser diode expressed by the term \( d\theta(t)/dt \) or \( d[\Delta\alpha(t)t]/dt \) in Eqs. (19) and (20). To remove the influence of these unwanted variations, we computed the ratio of \( \omega_o(t) \) to \( \omega_R(t) \). As shown in Fig. 7(b), the ratio takes a constant value of 0.5 despite the variations of \( \omega_o(t) \) and \( \omega_R(t) \). Noting that \( L_o = 50 \) cm and \( L_R = 100 \) cm, this result is in good agreement with that expressed in Eq. (21).

B. Three-Beam Interferometry

Instead of the Fizeau interferometer illustrated in Fig. 2, we used a Michelson interferometer with multiple arms as shown in Fig. 8. A collimated beam from the laser diode is first split by beam splitter BS1 into two beams, of which one is reflected by corner cube CC3, and the other is again split by BS2 into two beams directed toward corner cubes CC1 and CC2. Three beams reflected from the three corner cubes are combined to yield a fringe with three carrier-frequency components as expressed by Eq. (22) with

\[ N = 3. \]

Figure 9 shows part of a detected fringe signal after preprocessing. Note that the wave form is so different from a pure sinusoid that a conventional fringe-counting technique cannot be used. Again here we computed the Fourier transform of the fringe signal. Figure 10 shows the Fourier spectra of the fringe signal shown in Fig. 9. Three spectra are separated from each other by the carrier frequencies proportional to the optical path differences: \( L_{m} = 200 \) cm, \( L_{m} = 142 \) cm, and \( L_{m} = 58 \) cm. \( L_{m} \) stands for the optical path difference between the beams reflected from corner cubes CCm and CCn. The OFDR technique could be applied to these spectra by reading the positions of the spectrum peaks. However, it yields erroneous results because the positions of the spectrum peaks do not necessarily represent the carrier frequencies introduced by the optical path differences. This can be seen from the fact that each spectrum in Fig. 10 has an asymmetric shape because of the nonlinear and time-varying current-wavelength characteristic of the laser diode expressed by the term \( d\theta(t)/dt \) or \( d[\Delta\alpha(t)t]/dt \) in Eqs. (19) and (20). Furthermore the positions of the spectrum peaks generally fall between the discrete sample points in the frequency domain, so that we cannot even determine the spectrum peak positions simply by using the fast Fourier transform algorithm. To apply the FFT, three bandpass filters were used whose passbands are illustrated in Fig. 10. The instantaneous angular
frequencies $\omega_{13}(t)/\omega_{12}(t)$ and $\omega_{23}(t)/\omega_{12}(t)$, which give the ratios of the optical path differences $L_{13}/L_{12}$ and $L_{23}/L_{12}$, respectively. Figure 12(a) shows $\omega_{13}(t)/\omega_{12}(t)$, and Fig. 12(b) shows $\omega_{23}(t)/\omega_{12}(t)$. Note that the ratios remain constant despite the variations of the instantaneous frequencies, and their values are in agreement with those predicted from the given path differences. Since $L_{13} = L_{12} + L_{23}$, we can determine $L_{13}$ not only from the ratio of instantaneous frequencies $\omega_{13}(t)/\omega_{12}(t)$ but also from $\omega_{23}(t)/\omega_{12}(t)$ by

$$L_{12} = \left[1 + \frac{\omega_{23}(t)}{\omega_{12}(t)}\right] L_{13}. \quad (29)$$

Figure 12(c) shows the ratio of the optical path differences $L_{13}/L_{12}$ obtained from Eq. (29); it is in good agreement with Fig. 12(a). This redundancy can be used to reduce the influence of noise by averaging the two results. This is possible because the noise...

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**Fig. 7.** (a) Instantaneous angular frequencies $\omega_2(t)$ and $\omega_6(t)$; (b) ratio of $\omega_5(t)$ to $\omega_6(t)$.

**Fig. 9.** Part of the fringe signal obtained by the three-beam interferometer. The waveform is so different from a pure sinusoid that a conventional fringe-counting technique cannot be used.

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**Fig. 8.** Three-beam interferometer: LD, laser diode; CL, collimator lens; BS, beam splitter; CC, corner cube; AMP, amplifier; PD, photodetector; and LPF, low-pass filter.
included in separated spectrum passbands may be considered to be independent. Although we eliminated 100 data points from both ends of the data shown in Figs. 12(a) and 12(c), we still have $(1024 - 2 \times 100) \times 2 = 1648$ data for averaging, which will improve the signal-to-noise ratio by a factor of more than 40 for the independent noise assumed.

Finally, it should be noted that our three-beam interferometer system, shown in Fig. 8, is much simpler than the system with a pair of two-beam interferometers proposed by Kobayashi\(^5\) (shown in Fig. 4). Furthermore, we should also emphasize the importance of the role played by the FTT, since the conventional fringe-counting techniques cannot be applied to the three-beam interferometer system.

**VI. Discussion**

We briefly discuss some of the factors that affect the accuracy of measurement by the proposed technique. First, we focus on the nonlinear current-wavelength characteristic of the laser diode and show that its effect is most pronounced when we perform a high-speed measurement. Next, we discuss the influence of mechanical vibrations and show that it is the major source of the limitation of our technique.

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**A. Nonlinear Current-Wavelength Characteristic of the Laser Diode**

To examine the influence of the repetition frequency \(f_{LD}\) of the triangular ramp current applied to the laser diode, we conducted experiments for two-beam interferometry by changing the repetition frequency of the ramp current. Figures 13(a) and 13(b) show, respectively, fringe signals obtained for \(f_{LD} = 14.3\) and 143 Hz. A closer look at the pictures reveals that the fringes have a lower frequency for a few milliseconds after the folding points of the triangular ramp current. The fact that the same phenomenon can be observed irrespective of the direction of the current folding indicates that the nonlinearity of the laser diode is strongly time dependent. Figure 14 shows the
instantaneous frequencies $\omega(t)$ obtained for four different repetition frequencies. As seen in Fig. 14, the influence of the nonlinearity increases with the repetition frequency, and if the reference technique were not used, the error caused by the nonlinearity would become as large as 28% for the repetition frequency of 244 Hz. This means that combining the FTT with the reference technique is essential for high-speed measurements that require the laser diode to be driven by a ramp current with a high repetition frequency.

B. Influence of Mechanical Vibrations

The phase of the fringe signal is made of two components. Referring to Eq. (2), the first term $2\pi L/\lambda$ has a constant initial phase that commonly appears in conventional interferometry, and the second term $2\pi aL/\lambda^2$ is a time-varying phase introduced by the wavelength shift. Implicit in the proposed wavelength-shift interferometry is that the optical path difference $L$ is constant so that the phase variation is caused only by the second term. This, however, may not be the case in practical environments where mechanical vibrations and/or air turbulence are present. To discuss the influence of such mechanical vibrations and/or air turbulence, let us define the sensitivity of the phase to the variation of the optical path difference by the reciprocal of the effective wavelength. As in conventional interferometry, the first term $2\pi L/\lambda$ has the sensitivity $1/780$ (fringe/nm) given by the reciprocal of the wavelength $\lambda = 780$ nm of the laser diode. The second term $2\pi aL/\lambda^2$, on the other hand, has the sensitivity given by the reciprocal of the synthetic wavelength $\Lambda = \lambda^2/\Delta \lambda$, where $\Delta \lambda = \alpha t$. Since the maximum value of $\Delta \lambda$ is 0.06 nm in our experiments, the synthetic wavelength becomes $\Lambda = 10.14$ mm, which gives the sensitivity of the second term $1/\Lambda = 1/10.14$ (fringe/mm). Thus the sensitivity of the first term is 13,000 times larger than that of the second term. This means that, if the optical path length is changed by one wavelength, $\lambda = 780$ nm, by vibrations or air turbulence, the fringe shift introduced by it through the first term will give rise to a change in the measured distance by one synthetic wavelength $\Lambda = 10.14$ mm. This is because the fringe shift caused by the vibrations is interpreted as if it were introduced through the second term by the large movement of the object. This property of error magnification is the major source of limitation on the wavelength-shift interferometry applied to distance measurements. A laser diode whose wavelength can be scanned over a wider range without mode hops will reduce the error magnification. Another possibility is to reduce the effect of vibrations and/or air turbulence by taking the fringe data quickly for a short period during which the vibrations and/or air turbulence stay constant. For this quick data acquisition, a ramp current applied to the laser diode must have a high repetition frequency. As we have noted, the influence of the current-wavelength nonlinearity increases with the repetition frequency. Thus we again mention that combining the FTT with the reference technique is essential for practical measurements.

C. Separation of Frequency Spectra

In the principle described in Section IV, we have $1/2N(N - 1)$ frequency spectra on the positive-frequency axis, each corresponding to each positive-frequency term included in Eq. (25). These spectra are generated not only by the interference between the reference beam and each target beam but also by the interference between the target beams themselves. In our experimental demonstration, this was used to check the two measured distances [obtained from Eqs. (28) and (29)] against each other [see Figs. 12(a) and 12(c)]. However, this may cause problems
when the number of targets is increased. In general, the optical path differences between the targets may coincide with or become very close to those between the reference and the targets. The spectra corresponding to such optical path differences cannot be separated on the frequency axis. Furthermore, even when all the spectra are separated, it may become hard to distinguish the spectra corresponding to the optical path differences between the reference and the targets from those corresponding to the optical path differences between the targets themselves. Therefore, we may not be able to identify which spectrum corresponds to the optical path difference between the reference and the target of interest.

These problems will be solved or at least greatly alleviated if we make the relative intensity of the reference beam sufficiently larger than the intensities of the beams returning from the targets. This will cause the interference between the target beams to be negligibly small compared with that between the reference beam and the target beam. Thus Eq. (25) can be approximated by

\[
G(f) \approx A(f) + \sum_{\nu=1}^{N} \left[ C_{\nu}(f-f_{\nu}) + C_{\nu}^*\left[-(f+f_{\nu})\right] \right],
\]

where we have chosen the first mirror as a reference. In approximation (30) we have only \(N\) spectra [instead of \(\frac{1}{2}N(N-1)\) spectra] whose locations on the positive-frequency axis correspond to the optical path differences between the reference and targets. Thus we can identify each calculated distance to the corresponding object, and the spectrum overlapping can be avoided unless the targets are actually located close to each other.

VII. Conclusion

The FTT of the fringe signal analysis has been applied to wavelength-shift interferometry. We have shown how the FTT can be combined with the reference technique to remove the influence of the nonlinear and time-varying current-wavelength characteristic of the laser diode. We have proposed the principle of multiple-beam interferometry and have demonstrated distance measurements with a three-beam interferometer that includes a reference reflector as an integral part of the system to make it simpler than the existing system, which requires one extra interferometer for reference. We have discussed some of the error sources and clarified the limitation of the proposed technique.

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References