Digital Signal Processing

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Reference: Digital Signal Processing Laboratory Using Matlab
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Chapter 4 LTI Discrete-Time Signals in the Frequency domain

4.1 Background Review

C1. If $\{h[n]\}$ denotes the impulse response sequence of an LTI discrete-time system, its frequency response $H(e^{jw})$ is given by the discrete-time Fourier transform of $\{h[n]\}$, that is,

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n]e^{-jwn} \qquad (4.1)$$

C2. In general $H(e^{jw})$ is a complex function of w with a period 2π and can be expressed in terms of its real and imaginary parts or its magnitude and phase. Thus,

$$H(e^{jw}) = H_{re}(e^{jw}) + jH_{im}(e^{jw}) = \left|H(e^{jw})\right| e^{j\theta(w)} \qquad (4.2)$$

where $H_{re}(e^{jw})$ and $H_{im}(e^{jw})$ are, respectively, the real and imaginary parts of $H(e^{jw})$, and

$$\theta(w) = \arg\left\{H(e^{jw})\right\} \qquad (4.3)$$

The quantity $\left|H(e^{jw})\right|$ is called the magnitude response and the quantity of $\theta(w)$ is called the phase response of the LTI discrete-time response.

C3. The gain function $g(w)$ of the LTI system is defined by

$$g(w) = 20\ \log_{10}\left|H(e^{jw})\right|\ \text{dB} \qquad (4.4)$$

The negative of the gain function, $\alpha(w) = -g(w)$, is called the attenuation or loss function.

C4. The steady-state output $y[n]$ of a real coefficient LTI discrete-time system with a frequency response $H(e^{jw})$ for an input

$$x[n] = A\cos(w_0 n + \phi) \qquad (4.5)$$

with A real, is given by

$$y[n] = A\left|H(e^{jw_0})\right|\cos(w_0 n + \theta(w_0) + \phi) \qquad (4.6)$$

C5. The frequency response of an LTI discrete-time system is given by the ratio of the Fourier transform $Y(e^{jw})$ of the output sequence $y[n]$ to the Fourier transform $X(e^{jw})$ of the input sequence $x[n]$, that is,

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} \qquad (4.7)$$

C6. The z-transform $H(z)$ of the impulse response sequence $\{h[n]\}$ of the LTI discrete time system is called the transfer function or the system function. The transfer function $H(z)$ of an LTI discrete time system is given by the ratio of the z-transform $Y(z)$ of the output sequence $y[n]$ to the z-transform $X(z)$ of the input sequence $x[n]$; that is, $H(z) = Y(z)/X(z)$.

C7. If the ROC of $H(z)$ includes the unit circle, it is then related to the frequency response $H(e^{jw})$ of the LTI discrete-time system through

$$H(e^{jw}) = H(z)\big|_{z=e^{jw}} \qquad (4.8)$$

C8. For a real-coefficient transfer function $H(z)$:

$$\left|H(e^{jw})\right|^2 = H(e^{jw})H^*(e^{jw}) = H(e^{jw})H(e^{-jw}) = H(z)H(z^{-1})\Big|_{z=e^{jw}} \qquad (4.9)$$

C9. For an LTI system characterized by a linear constant-coefficient difference equation of the form of Eq. (2.11), the transfer function $H(z)$ can be expressed as:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{p_0 + p_1 z^{-1} + \cdots + p_M z^{-M}}{d_0 + d_1 z^{-1} + \cdots + d_N z^{-N}} = \frac{p_0}{d_0}\frac{\prod_{k=1}^{M}(1-\xi_k z^{-1})}{\prod_{k=1}^{N}(1-\lambda_k z^{-1})} \qquad (4.10)$$

where $\xi_1, \xi_2, \cdots, \xi_M$ are the finite zeros and $\lambda_1, \lambda_2, \cdots, \lambda_N$ are the finite poles of $H(z)$. If $N > M$, there are additional $(N - M)$ zeros at $z = 0$, and If $N < M$, there are additional $(M - N)$ poles at $z = 0$.

C10. All poles of a stable causal transfer function $H(z)$ must be strictly inside the unit circle.

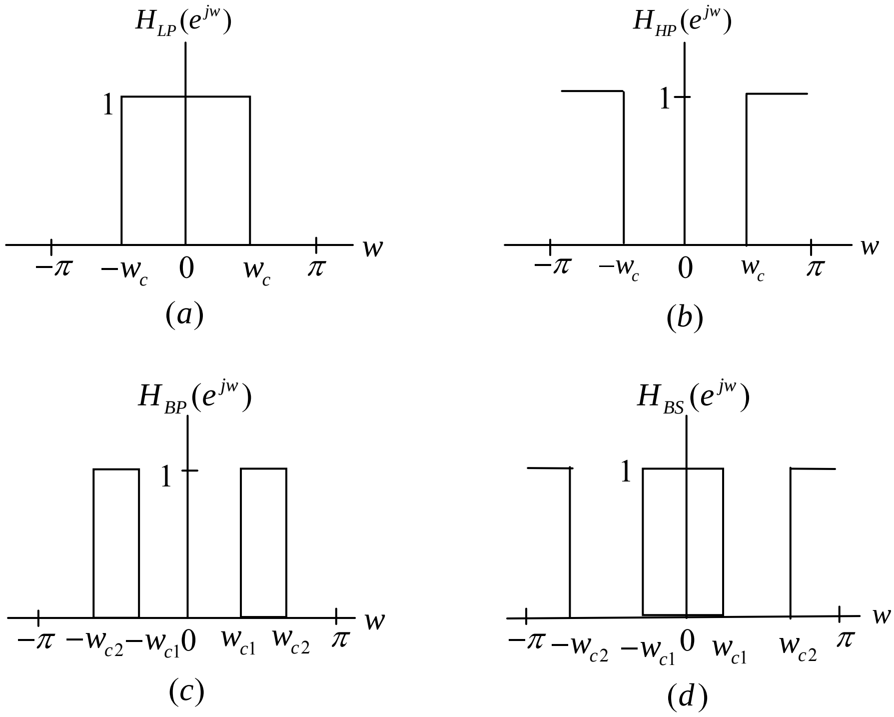


Figure 1 Frequency responses of ideal filters (a) lowpass (b) highpass (c) bandpass (d) bandstop

C11. The impulse response $h_{LP}[n]$ of the ideal lowpass filter of Figure 1 is given by

$$h_{LP}[n] = \frac{\sin(w_c n)}{\pi n}, \quad -\infty < n < \infty \qquad (4.11)$$

C12. A first-order lowpass IIR transfer function $H_{LP}(z)$ is given by

$$H_{LP}(z) = \frac{1-\alpha}{2}\frac{1+z^{-1}}{1-\alpha z^{-1}} \qquad (4.12)$$

where $|\alpha| < 1$ for stability. The frequency w_c where the gain is 3dB below its maximum value at dc ($w = 0$), called the 3-dB cutoff frequency, is related to the parameter α through

$$\alpha = \frac{1-\sin w_c}{\cos w_c} \qquad (4.13)$$

C13. A first-order highpass IIR transfer function $H_{HP}(z)$ is given by

$$H_{HP}(z) = \frac{1+\alpha}{2} \frac{1-z^{-1}}{1-\alpha z^{-1}} \quad (4.14)$$

where $|\alpha| < 1$ for stability. Its 3-dB cutoff frequency w_c is also given by Eq. (4.13)

C14. A second-order bandpass IIR transfer function $H_{BP}(z)$ is given by

$$H_{BP}(z) = \frac{1-\alpha}{2} \frac{1-z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}} \quad (4.15)$$

Its magnitude response goes to zero at $w = 0$ and $w = \pi$ and assumes a maximum value of unity $w = w_0$ called the center frequency of the bandpass filter, where

$$w_0 = \cos^{-1}(\beta) \quad (4.16)$$

The 3-dB bandwidth $\Delta w_{3\text{dB}}$, given by the difference of the two 3-dB cutoff frequency, is given by

$$\Delta w_{3\text{dB}} = w_{c2} - w_{c1} = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) \quad (4.17)$$

C15. A second-order bandstop IIR transfer function $H_{BS}(z)$ is given by

$$H_{BS}(z) = \frac{1+\alpha}{2} \frac{1-2\beta z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}} \quad (4.18)$$

Its magnitude response takes the maximum value of unity at $w = 0$ and $w = \pi$ and goes to zero at $w = w_0$, called the notch frequency, where w_0 is given by Eq. (4.16). The 3-dB notch bandwidth $\Delta w_{3\text{dB}}$ is given by Eq. (4.17).

C16. By cascading the simple digital filters described above, digital filters with sharper magnitude responses can be implemented. For example, for a cascade of K first-order lowpass sections characterized by the transfer function of Eq. (4.12), the overall structure has a transfer function $G_{LP}(z)$ given by

$$G_{LP}(z) = \left(\frac{1-\alpha}{2} \frac{1+z^{-1}}{1-\alpha z^{-1}}\right)^K \quad (4.19)$$

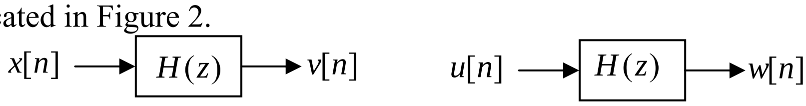
The parameters α *and* K are related to the 3-dB cutoff frequency w_c of the cascade through

$$\alpha = \frac{1+(1-B)\cos w_c - \sin w_c \sqrt{2B-B^2}}{1-B+\cos w_c} \quad (4.20)$$

Where

$$B = 2^{(K-1)/K} \quad (4.21)$$

C17. For non-real time processing of input signals of finite length, zero-phase filtering can be very simply implemented if the causality requirement is relaxed. In one scheme, the finite-length input data are processed through a causal real-coefficient filter $H(z)$ whose output is then time-reversed and processed by the same filter once again as indicated in Figure 2.



$$u[n] = v[-n],\ y[n] = w[-n]$$

Figure 2 Implementation of a zero-phase filtering scheme.

C18. It is always possible to design an FIR transfer function with an exact linear phase response. Such a transfer function corresponds either to a symmetric impulse response defined by

$$h[n] = h[N-n], \quad 0 \le n \le N \tag{4.22}$$

or an antisymmetric impulse response defined by

$$h[n] = -h[N-n], \quad 0 \le n \le N \tag{4.23}$$

where N is the order of the transfer function and the length of $h[n]$ is $N + 1$. There are four types of such transfer functions:

Type 1: Symmetric Impulse Response with Odd Length.
Type 2: Symmetric Impulse Response with Even Length.
Type 3: Antisymmetric Impulse Response with Odd Length.
Type 4: Antisymmetric Impulse Response with Even Length.

C19. A Type 2 FIR transfer function must have a zero at $z = -1$, and as a result, it cannot be used to design a highpass filter. A Type 3 FIR transfer function must have a zero at $z = 1$ and $z = -1$, and therefore, cannot be used to design either a lowpass, a highpass, or a bandstop filter. A Type 4 FIR transfer function is not appropriate for designing a lowpass filter due to the presence of a zero at $z = 1$. The Type 1 FIR filter has no such restrictions and can be used to design almost any type of filter.

C20. A causal stable real coefficient transfer function $H(z)$ is defined as a bounded real (BR) transfer function if

$$\left|H(e^{jw})\right| \le 1 \quad \textit{for all } w. \tag{4.24}$$

C21. A transfer function $H(z)$ with unity magnitude response for all frequencies, that is,

$$\left|H(e^{jw})\right|^2 = 1 \quad \textit{for all } w, \tag{4.25}$$

Is called an allpass transfer function. An *M-th* order causal real coefficient IIR allpass transfer function is of the form

$$H(z) = \pm \frac{z^{-M} D_M(z^{-1})}{D_M(z)}, \tag{4.26}$$

where $D_M(z)$ is a polynomial of degree M. The poles and zeros of a real-coefficient allpass function exhibit mirror-image symmetry in the z-plane. If the allpass transfer function is also causal and stable, then all its poles are inside the unit circle and all its zeros are outside the unit circle in a mirror-image symmetry with the poles.

C22. A causal stable transfer function with all zeros inside or on the unit circle is called a minimum-phase transfer function, whereas a causal stable transfer function with all zeros outside the unit circle is called a maximum-phase transfer function.

C23. A set of M transfer functions $\{H_0(z), H_1(z), \ldots, H_{M-1}(z)\}$ is defined to be delay-complementary of each other, if the sum of their transfer functions is equal to some integer multiple of the unit delay, that is,

$$\sum_{k=0}^{M-1} H_k(z) = \beta z^{-n_0}, \quad \beta \ne 0 \tag{4.27}$$

where n_0 is a nonnegative integer. The delay-complementary transfer function $H_1(z)$ to a Type 1 linear-phase FIR transfer function $H_0(z)$ of odd length L is simply given by $H_1(z) = z^{-(L-1)/2} - H_0(z)$.

C24. A set of M digital filters $\{H_i(z)\}$, $i = 0, 1, \ldots, M-1$, are defined to be allpass-complementary of each other if the sum of their transfer functions is equal to an allpass function $A(z)$, that is,

$$\sum_{i=0}^{M-1} H_i(z) = A(z). \tag{4.28}$$

C25. A set of M digital filters $\{H_i(z)\}, \quad i = 0,\ 1,\ \ldots,\ M-1,$ are defined to be power-complementary of each other if the sum of the squares of their magnitude responses is equal to one, that is,

$$\sum_{i=0}^{M-1} \left|H_i(e^{jw})\right|^2 = 1 \qquad \textit{for all } w \tag{4.29}$$

C26. Let $A_m(z)$ be a real-coefficient allpass function of *mth* order:

$$A_m(z) = \frac{d_m + d_{m-1}z^{-1} + d_{m-2}z^{-2} + \ldots + d_1 z^{-(m-1)} + z^{-m}}{1 + d_1 z^{-1} + d_2 z^{-2} + \ldots + d_{m-1}z^{-(m-1)} + d_m z^{-m}} \tag{4.30}$$

Generate an (m - 1)th order real-coefficient allpass function $A_{m-1}(z)$ according to

$$\begin{aligned} A_{m-1}(z) &= z\left[\frac{A_m(z) - k_m}{1 - k_m A_m(z)}\right] \\ &= \frac{d'_{m-1} + d'_{m-2}z^{-1} + \ldots + d'_1 z^{-(m-2)} + z^{-(m-1)}}{1 + d'_1 z^{-1} + \ldots + d'_{m-2}z^{-(m-2)} + d'_{m-1}z^{-(m-1)}} \end{aligned} \tag{4.31}$$

where

$$d'_i = \frac{d_i - d_m d_{m-i}}{1 - d_m^2}, \qquad i = 1,\ 2, \ldots,\ m-1. \tag{4.32}$$

Define $k_m = A_m(\infty)$. The necessary and sufficient conditions for $A_m(z)$ to be stable are (1) $k_m^2 < 1$ and (2) $A_{m-1}(z)$ is a stable allpass function. The process can be continued to test the stability of $A_{m-1}(z)$ by generating an (m - 2)th order allpass function, and so on, resulting in a set of allpass functions of decreasing orders

$$A_m(z),\ A_{m-1}(z),\ \ldots,\ A_2(z),\ A_1(z),\ A_0(z) = 1,$$

and a set of coefficients

$$k_m,\ k_{m-1},\ \ldots,\ k_2,\ k_1.$$

The allpass function $A_m(z)$ is stable if and only if $k_l^2 < 1$ for $l = m,\ m-1,\ \ldots,\ 1.$

4.2 Transfer Function and Frequency Response

Project 1: Transfer function Analysis

Program P3_1:

```
% Program P3_1
% Evaluation of the DTFT
clf;
% Compute the frequency samples of the DTFT
w = -4*pi:8*pi/511:4*pi;
num = [2 1];den = [1 -0.6];
h = freqz(num, den, w);
% Plot the DTFT
subplot(2,1,1)
plot(w/pi,real(h));grid
title('Real part of H(e^{j\omega})')
xlabel('\omega /\pi');
```

```
ylabel('Amplitude');
subplot(2,1,2)
plot(w/pi,imag(h));grid
title('Imaginary part of H(e^{j\omega})')
xlabel('\omega /\pi');
ylabel('Amplitude');
pause
subplot(2,1,1)
plot(w/pi,abs(h));grid
title('Magnitude Spectrum |H(e^{j\omega})|')
xlabel('\omega /\pi');
ylabel('Amplitude');
subplot(2,1,2)
plot(w/pi,angle(h));grid
title('Phase Spectrum arg[H(e^{j\omega})]')
xlabel('\omega /\pi');
ylabel('Phase in radians');
```

Questions:

Q1 Modify Program P3_1 to compute and plot the magnitude and phase spectra of a moving filter of Eq. $y[n]=\frac{1}{M}\sum_{k=0}^{M-1}x[n-k]$ for three different values of length M and for $0 \le w \le 2\pi$ .Justify the type of symmetries exhibited by the magnitude and phase spectra. What type of filter does it represent?

Q2 Using the modified Program P3_1 compute and plot the frequency response of a causal LTI discrete-time system with a transfer function given by

$$H(z)=\frac{0.15(1-z^{-2})}{1-0.5z^{-1}+0.7z^{-2}} \qquad (4.33)$$

for $0 \le w \le \pi$ .What type of filter does represent?

Q3 Repeat Q2 for the following transfer function:

$$G(z)=\frac{0.15(1-z^{-2})}{0.7-0.5z^{-1}+z^{-2}} \qquad (4.34)$$

What is the difference between the two filters of Eq. (4.33) and (4.34)? Which one will you choose for filtering and why

Q4 Using Matlab command grpdelay compute and plot the group delay of the causal LTI discrete-time system with a transfer function given by

$$H(z)=\frac{z^{-1}-1.2z^{-2}+z^{-3}}{1-1.3z^{-1}+1.04z^{-2}-0.222z^{-3}} \qquad (4.35)$$

for $0 \le w \le \pi$.

Q5 Using Matlab command zplane develop the pole-zero plots of the two filters of Eq. (4.33) and (4.34), respectively. Comment on your results.

4.3 Types Transfer Functions

Project 2: Filters

The impulse response $h_{LP}[n]$ of the ideal lowpass filter of Figure 1 given by Eq. (4.11) is doubly infinite and cannot be implemented. Hence a simple approximation is achieved by just truncating the impulse response to a finite number terms. However, the truncated impulse response represents a non-causal filter. A causal approximation is then obtained by shifting the truncated filter impulse response to the right by $N/2$ samples, resulting in

$$\hat{h}_{LP}[n] = \frac{\sin w_c(n - N/2)}{\pi(n - N/2)}, \qquad 0 \le n \le N. \tag{4.36}$$

The length of the filter is $N + 1$.

Program P4_1: Impulse Response of **Truncated Ideal Lowpass** Filter

```
% Program P4_1
% Impulse Response of Truncated Ideal Lowpass Filter
clf;
fc = 0.25;
n = [-6.5:1:6.5];
y = 2*fc*sinc(2*fc*n);k = n+6.5;
stem(k,y);title('N = 13');axis([0 13 -0.2 0.6]);
xlabel('Time index n');ylabel('Amplitude');grid;
```

Questions:

Q6 Compute and plot the impulse response of the approximation to the ideal lowpass filter using Program P4_1. What is the length of the FIR lowpass filter? Which statement in Program P4_1 determines the filter length? Which parameter controls the cutoff frequency?

Q7 Modify Program P4_1 to compute and plot the impulse response of the FIR lowpass filter of Eq. (4.36) with a length of 20 and an angular cutoff frequency of $w_c = 0.45$.

Q8 Modify Program P4_1 to compute and plot the impulse response of the FIR lowpass filter of Eq. (4.36) with a length of 15 and an angular cutoff frequency of $w_c = 0.65$

The moving-average filter $y[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]$ also has a lowpass magnitude response as seen from the results of Question Q1. The simplest such filter is of length 2 and has a transfer function

$$H_0 = \frac{1}{2}(1 + z^{-1}) \tag{4.37}$$

It can be shown that this filter has a 3-dB cutoff frequency $w_c = \pi/2$. By cascading a number of these simple FIR lowpass filters, a lowpass filter with a sharper magnitude response can be obtained. A cascade of K sections of $H_0(z)$ has a 3-dB cutoff frequency at

$$w_c = 2\cos^{-1}(2^{-1/2K}) \qquad (4.38)$$

A slight modification of the difference equation $y[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]$ yields a high pass filter whose transfer function is given by

$$H_1(z) = \frac{1}{M}\sum_{k=0}^{M-1}(-1)^n z^{-n} \qquad (4.39)$$

The function gain given below computes and plots the gain response in dB of a rational transfer function.

```
function [g,w] = gain(num,den)
% Computes the gain function in dB of a
% transfer function at 256 equally spaced points
% on the top half of the unit circle
% Numerator coefficients are in vector num
% Denominator coefficients are in vector den
% Frequency values are returned in vector w
% Gain values are returned in vector g
w = 0:pi/255:pi;
h = freqz(num,den,w);
g = 20*log10(abs(h));
```

Program P4_2:

```
% Program P4_2
% Gain Response of a Moving Average Lowpass Filter
clf;
M = 2;
num = ones(1,M)/M;
[g,w] = gain(num,1);
plot(w/pi,g);grid
axis([0 1 -50 0.5])
xlabel('\omega /\pi');ylabel('Gain in dB');
title(['M = ', num2str(M)])
```

Questions:

Q9 Run Program P4_2 to compute and plot the gain response of a length-2 moving average filter. From the plot verify that the 3-dB cutoff frequency is at $w_c = \pi/2$.

Q10 Modify Program P4_2 to compute and plot the gain response of a cascade of K length-2 moving average filters. Using the modified program plot the gain response for a cascade of 3 sections and verifies that the 3-dB cutoff frequency of the cascade is as given by Eq. (4.38).

Q11 Modify Program P4_2 to compute and plot the gain response of the highpass filter of Eq. (4.39). Run the modified program to plot the gain response for M = 5 and determine its 3-dB cutoff frequency from the plot.

In many applications, the simple first-order and second-order IIR filters described in C12 through C15 are adequate.

Questions:

Q12 Design a first-order IIR lowpass and a first-order IIR highpass filter with a 3-dB angular cutoff frequency $w_c = 0.45\pi$. Using MATLAB compute and plot their gain responses, and verify that the designed filters meet the specification.

Q13 Design an IIR lowpass filter with a 3-dB cutoff frequency $w_c = 0.3\pi$ by cascading 10 sections of the first-order IIR lowpass filter of Eq. (4.12). Compare its gain response with that of a first-order IIR lowpass filter designed for the same cutoff frequency.

Q14 Design a second-order IIR bandpass filter with a center frequency $w_o = 0.61\pi$ and a 3-dB bandwidth of 0.15π.

If $H(z)$ is the transfer function of an FIR or IIR digital filter, the filter obtained by replacing each delay in the realization of $H(z)$ by L delays has a transfer function $G(z) = H(z^L)$. Thus, the new filter has a frequency response that is a periodic function of w with a period $2\pi / L$. Such filters are generally called **comb filters** and find applications in rejecting periodic interferences.

Questions:

Q15 Using MATLAB compute and plot the magnitude response of a comb filter obtained from a prototype FIR lowpass filter of Eq. (4.35) for different values of L. Show that the new filter has multiple notches at $w = w_k = (2k+1)\pi / L$ and has L peaks in its magnitude response at $w = w_k = 2k\pi / L,\ k = 0,\ 1,\ \cdots,\ L\text{-}1$.

Q16 Using MATLAB compute and plot the magnitude response of a comb filter obtained from a prototype FIR highpass filter of Eq. (4.38) with $M = 2$ and for different values of L. Determine the locations of the notches and the peaks of the magnitude response of this type of comb filter.

Many applications require the use of digital filters with either linear phase or zero phase. Zero phase filtering cannot be implemented using a causal digital filter. However the non-causal implementation indicated in Figure 2 can be employed for zero-phase filtering using either an FIR or an IIR digital filter. The MATAB M-file filtfilt implements this type of zero-phase filtering scheme. Four types of linear phase FIR filters are defined (see C17). Program P4_3 can be used to investigate the properties of these filters. Its first generates the plots of the impulse response sequence of each of the four types, then generates the pole-zero plots, and finally displays the zero locations.

Program P4_3

```
% Zero Locations of Linear Phase FIR Filters
clf;
b = [1 -8.5 30.5 -63];
num1 = [b 81 fliplr(b)];
num2 = [b 81 81 fliplr(b)];
num3 = [b 0 -fliplr(b)];
num4 = [b 81 -81 -fliplr(b)];
n1 = 0:length(num1)-1;
n2 = 0:length(num2)-1;
subplot(2,2,1); stem(n1,num1);
xlabel('Time index n');ylabel('Amplitude'); grid;
title('Type 1 FIR Filter');
subplot(2,2,2); stem(n2,num2);
xlabel('Time index n');ylabel('Amplitude'); grid;
title('Type 2 FIR Filter');
subplot(2,2,3); stem(n1,num3);
xlabel('Time index n');ylabel('Amplitude'); grid;
title('Type 3 FIR Filter');
subplot(2,2,4); stem(n2,num4);
xlabel('Time index n');ylabel('Amplitude'); grid;
title('Type 4 FIR Filter');
pause
subplot(2,2,1); zplane(num1,1);
title('Type 1 FIR Filter');
subplot(2,2,2); zplane(num2,1);
title('Type 2 FIR Filter');
subplot(2,2,3); zplane(num3,1);
title('Type 3 FIR Filter');
subplot(2,2,4); zplane(num4,1);
title('Type 4 FIR Filter');
disp('Zeros of Type 1 FIR Filter are');
```

```
disp(roots(num1));
disp('Zeros of Type 2 FIR Filter are');
disp(roots(num2));
disp('Zeros of Type 3 FIR Filter are');
disp(roots(num3));
disp('Zeros of Type 4 FIR Filter are');
disp(roots(num4));
```

Questions:

Q17 In Program P4_3, what are the lengths of each FIR filter? Verify the symmetry properties of the impulse response sequences.

Q18 Replace the vector b in Program P4_3 with b = [1.5 -3.25 5.25 -4] and repeat Q17.

4.4 Stability Test

The stability of an IIR causal digital filter is an important design requirement. A causal IIR filter is stable if all poles of its transfer function are inside the unit circle. The MATLAB function zplane can be used to check the pole locations of an IIR transfer function. However, if one or more poles are very close to the unit circle or on the unit circle, the pole-zero plot is not sufficient to test the stability of the corresponding transfer function. A more accurate stability test is based on the algorithm given in C26. Program P4_4 implements this using the function poly2rc.

Program P4_4

```
% Stability Test
clf;
den = input('Denominator coefficients = '); % form den = [1 -1.848 0.85];
ki = poly2rc(den);
disp('Stability test parameters are');
disp(ki);
```

Questions:

Q19 Using MATLAB generates the pole-zero plots of the following two causal transfer functions.

$$H_1(z) = \frac{1}{1\text{-}1.848z^{-1} + 0.85z^{-2}}$$

$$H_2(z) = \frac{1}{1\text{-}1.851z^{-1} + 0.85z^{-2}}$$

Q20 Test the stability of the two transfer function in Q19 using Program P4_4. Which one of the two transfer functions is stable?