Digital Signal Processing

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Reference: Digital Signal Processing Laboratory Using Matlab
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Chapter 3 Discrete-Time Signals in the Frequency domain

3.1 Background Review

C1. The discrete-time Fourier transform (DTFT) $X(e^{jw})$ of a sequence $x[n]$ is defined by

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn} \tag{3.1}$$

In general $X(e^{jw})$ is a complex function of the real variable *w* and can be written as

$$X(e^{jw}) = X_{re}(e^{jw}) + jX_{im}(e^{jw}) \tag{3.2}$$

where $X_{re}(e^{jw})$ and $X_{im}(e^{jw})$ are, respectively, the real and imaginary parts of $X(e^{jw})$, and are real functions of *w*. $X(e^{jw})$ can alternately be expressed in the form

$$X(e^{jw}) = \left|X(e^{jw})\right| e^{j\theta(w)} \tag{3.3}$$

where

$$\theta(w) = \arg\left\{X(e^{jw})\right\} \tag{3.4}$$

The quantity $\left|X(e^{jw})\right|$ is called the magnitude function and the quantity of $\theta(w)$ is called the phase function, with both functions again being real functions of *w*. In many applications, the Fourier transform is called the Fourier spectrum and, likewise, $\left|X(e^{jw})\right|$ and $\theta(w)$ are referred to as the magnitude spectrum and phase spectrum, respectively.

C2. The DTFT $X(e^{jw})$ is a periodic continuous function in *w* with a period 2π.

C3. For a real sequence $x[n]$, the real part $X_{re}(e^{jw})$ of its DFTF and the magnitude function $\left|X(e^{jw})\right|$ are even functions of *w*, whereas the imaginary part $X_{im}(e^{jw})$ and the phase function $\theta(w)$ are odd functions of *w*.

C4. The **inverse discrete-time Fourier transform** $x[n]$ of $X(e^{jw})$ is given by

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})dw \tag{3.5}$$

C5. The Fourier transform $X(e^{jw})$ of a sequence $x[n]$ exists if $x[n]$ is **absolutely summable**, that is,

$$\sum_{n=-\infty}^{\infty} \left|x[n]\right| < \infty \tag{3.6}$$

C6. Some properties of DTFT

a. **Time-Shifting Property:**

If $X(e^{jw})$ denotes the DTFT of a sequence $x[n]$, then the DTFT of the time shifted sequence $x[n-n_0]$ is given by $e^{-jwn_0}X(e^{jw})$.

b. **Frequency-Shifting Property:**

If $X(e^{jw})$ denotes the DTFT of a sequence $x[n]$, then the DTFT of the $e^{jw_0 n}x[n]$ is given by $X(e^{j(w-w_0)})$.

c. **Convolution Property:**

If $X(e^{jw})$ and $Y(e^{jw})$ denote the DTFT of sequences $x[n]$ and $y[n]$, respectively, then the DTFT of the sequence $x[n]*y[n]$ is given by $X(e^{jw})Y(e^{jw})$.

d. Modulation Property:

If $X(e^{jw})$ and $Y(e^{jw})$ denote the DTFT of sequences $x[n]$ and $y[n]$, respectively, then the DTFT of the sequence $x[n]y[n]$ is given by

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(w-\theta)})d\theta$$

e. **Time Reversal Property:**

If $X(e^{jw})$ denotes the DTFT of a sequence $x[n]$, then the DTFT of the time reversed sequence $x[-n]$ is given by $X(e^{-jw})$.

C7. The N-point discrete Fourier transform(DFT) of a finite length sequence $x[n]$, defined for $0 \le n \le N-1$, is given by

$$X[k] = \sum_{n=0}^{N-1} x[n]W_N^{kn}, \quad k = 0,\ 1,\ \cdots,\ N-1 \tag{3.7}$$

where

$$W_N = e^{-j2\pi/N} \tag{3.8}$$

C8. The N-point discrete Fourier transform(DFT) of a finite length sequence $x[n]$, $n = 0,\ 1,\ \cdots,\ N-1$, is simply the frequency samples of its DTFT $X(e^{jw})$ evaluated at N uniformly spaced frequency points, $w = w_k = 2\pi k/N$, $k = 0,\ 1,\ \cdots,\ N-1$, that is,

$$X[k] = X(e^{jw})\Big|_{w=2\pi k/N}, \qquad k = 0,\ 1,\ \cdots,\ N-1 \tag{3.9}$$

C9. The N-point circular convolution of two length N sequences $g[n]$ and $h[n]$, $0 \le n \le N-1$, is defined by

$$y_c[n] = \sum_{m=0}^{N-1} g[m]h[\langle n-m\rangle_N] \tag{3.10}$$

where $\langle n\rangle_N = n$ modulo N. The N-point circular convolution operation is usually denoted as

$$y_c[n] = g[n] \circledN h[n] \tag{3.11}$$

C10. The linear convolution of a length-*N* sequence $g[n],\ 0 \le n \le N-1,$ with a length-*M* sequence $h[n],\ 0 \le n \le M-1,$ can be obtained by a (*N*+*M*-*1*)-point circular convolution of two length-(*N*+*M*-*1*) sequences, $g_e[n]$ and $h_e[n]$,

$$y_L[n] = g[n] \circledast h[n] = g_e[n] \circledN h_e[n], \tag{3.12}$$

where $g_e[n]$ and $h_e[n]$ are obtained by appending $g[n]$ and $h[n]$ with zero-valued samples:

$$g_e[n] = \begin{cases} g[n], & 0 \le n \le N\text{-}1 \\ 0, & N \le n \le N+M-1 \end{cases} \tag{3.13}$$

$$h_e[n] = \begin{cases} h[n], & 0 \le n \le M\text{-}1 \\ 0, & M \le n \le N+M-1 \end{cases} \tag{3.14}$$

C11. Some properties of DFT

Circular Time-Shifting Property:

If $G[k]$ denotes the *N*-point DFT of a length-*N* sequence $g[n]$, then the *N*-point DFT of the circularly time-shifted sequence $g\left[\langle n-n_0\rangle_N\right]$ is given by $W_N^{kn_0}G[k]$ where $W_N = e^{-j2\pi/N}$.

Circular Frequency-Shifting Property:

If $G[k]$ denotes the *N*-point DFT of a length-*N* sequence $g[n]$, then the *N*-point DFT of the sequence $W_N^{-k_0 n}g[n]$ is given by $G\left[\langle k-k_0\rangle_N\right]$.

Circular Convolution Property:

If $G[k]$ and $H[k]$ denote the *N*-point DFTs of the length-*N* sequences $g[n]$ and $h[n]$, respectively, then the *N*-point DFT of the circularly convolved sequence is given by $G[k]H[k]$.

Parseval's Relation:

If $G[k]$ denotes the *N*-point DFT of a length-*N* sequence $g[n]$, then

$$\frac{1}{N}\sum_{n=0}^{N-1}\left|g[n]\right|^2 = \sum_{k=0}^{N-1}\left|G[k]\right|^2 \tag{3.15}$$

C12. The periodic even part $g_{pe}[n]$ and the periodic odd part $g_{po}[n]$ of a length-*N* real sequence $g[n]$ are given by

$$g_{pe}[n] = \frac{1}{2}\left(g[n] + g\left[\langle -n\rangle_N\right]\right) \tag{3.16}$$

$$g_{po}[n] = \frac{1}{2}\left(g[n] - g\left[\langle -n\rangle_N\right]\right) \tag{3.17}$$

If $G[k]$ denotes the *N*-point DFT of $g[n]$, then the *N*-point DFTs of $g_{pe}[n]$ and $g_{po}[n]$ are given by $\mathrm{Re}\{G[k]\}$ *and* $j\,\mathrm{Im}\{G[k]\}$, respectively.

C13. Let $g[n]$ *and* $h[n]$ be two length-*N* real sequences, with $G[k]$ and $H[k]$ denoting their respective N-point DFTs $X[k]$ of a complex length-N sequence $x[n]$ defined by $x[n] = g[n] + jh[n]$ using

$$G[k] = \frac{1}{2}\left(X\left[k\right] + X^*\left[\langle -k \rangle_N\right]\right) \tag{3.18}$$

$$H[k] = \frac{1}{2j}\left(X\left[k\right] - X^*\left[\langle -k \rangle_N\right]\right) \tag{3.19}$$

C14. Let $v[n]$ be a real sequence of length $2N$ with $V[k]$ denoting its $2N$-point DFT. Define two real sequences $g[n]$ and $h[n]$ of length N each as

$$g[n] = v[2n] \; and \; h[n] = v[2n+1], \quad 0 \le n < N \tag{3.20}$$

with $G[k]$ and $H[k]$ denoting their N-point DFTs. Then the $2N$-point DFT $V[k]$ of $v[n]$ can be computed from the two N-point DFTs, $G[k]$ and $H[k]$, using

$$V\left[k\right] = G\left[\langle k \rangle_N\right] + W_{2N}^{k} H\left[\langle k \rangle_N\right], \quad 0 \le n < N \tag{3.21}$$

C15. The z-transform $G[z]$ of a sequence $g[n]$ is defined as

$$G[z] = Z\left\{g[n]\right\} = \sum_{n=-\infty}^{\infty} g[n] z^{-n} \tag{3.22}$$

where z is a complex variable. The set R of values of z for which the z-transform $G[z]$ converges is called its region of convergence (ROC). In general, the region of convergence R of a z-transform of a sequence $g[n]$ ia an annular region of the z-plane:

$$R_{g^-} < |z| < R_{g^+} \tag{3.23}$$

where $0 \le R_{g^-} < R_{g^+} \le \infty$.

C16. In the case of LTI discrete-time systems, all pertinent z-transforms are rational functions of z^{-1}, that is, they are ratios of two polynomials in z^{-1}

$$G[z] = \frac{P(z)}{D(z)} = \frac{p_0 + p_1 z^{-1} + \cdots + p_{M-1} z^{-(M-1)} + p_M z^{-M}}{d_0 + d_1 z^{-1} + \cdots + d_{N-1} z^{-(N-1)} + d_N z^{-N}} \tag{3.24}$$

which can be alternately written in factored form as

$$G[z] = \frac{p_0}{d_0} \frac{\prod_{r=1}^{M} (1 - \xi_r z^{-1})}{\prod_{s=1}^{N} (1 - \lambda_s z^{-1})} = \frac{p_0}{d_0} z^{N-M} \frac{\prod_{r=1}^{M} (z - \xi_r)}{\prod_{s=1}^{N} (z - \lambda_s)} \tag{3.25}$$

The zeros of $G[z]$ are given by $z = \xi_r$ whiles the poles are given by $z = \lambda_s$. There are additional $(N - M)$ zeros at $z = 0$ (the origin in the z-plane) if $N > M$ or additional $(M - N)$ zeros at $z = 0$ (the origin in the z-plane) if $N < M$.

C17. For a sequence with a rational z-transform, the ROC of the z-transform cannot contain any poles and is bounded by the poles.

C18. The inverse z-transform $g[n]$ of a z-transform $G[z]$ is given by

$$g[n] = \frac{1}{2\pi j}\oint_C G(z) z^{n-1} dz \tag{3.26}$$

where C is a counterclockwise contour encircling the point $z = 0$ in the ROC of $G(z)$.

C19. A rational z-transform $G[z] = P(z)/D(z)$, where the degree of the polynomial $P(z)$ is M and the degree of the polynomial $D(z)$ is N, and with distinct poles at $z = \lambda_s$, $s = 1, 2, \ldots, N$, can be expressed in a partial-fraction expansion form given by

$$G(z) = \sum_{l=0}^{M-N} \eta_l z^{-l} + \sum_{s=0}^{N} \frac{\rho_s}{1 - \lambda_s z^{-1}} \tag{3.27}$$

assuming $M \geq N$. The constants ρ_s in the above expression, called the residues, are given by

$$\rho_s = (1 - \lambda_s - z^{-1}) G(z)\Big|_{z=\lambda_s} \tag{3.28}$$

If $G(z)$ has multiple poles, the partial-fraction expansion is of slightly different form. For example, if the pole at $z = v$ is of multiplicity L and the remaining $N - L$ poles are simple and at $z = \lambda_s$, $s = 1, 2, \ldots, N - L$, then the general partial fraction expansion of $G(z)$ takes the form

$$G(z) = \sum_{l=0}^{M-N} \eta_l z^{-l} + \sum_{s=0}^{N-L} \frac{\rho_s}{1 - \lambda_s z^{-1}} + \sum_{r=1}^{L} \frac{\gamma_r}{(1 - v z^{-1})^r} \tag{3.29}$$

Where the constants γ_r (no longer called residues for $r \neq 1$) are computed using the formula

$$\gamma_r = \frac{1}{(L-r)!(-v)^{L-r}} \frac{d^{L-r}}{d(z^{-1})^{L-r}} \left[\left(1 - vz^{-1}\right)^L G(z) \right]_{z=v}, \quad r = 1, \ldots, L \tag{3.30}$$

and the residues ρ_s are calculated using Eq. (3.28)

3.2 Discrete-time Fourier Transform

The discrete-time Fourier Transform (DTFT) of a sequence $x[n]$ is a continuous function of w. Since the data in MATLAB is in vector form, $X(e^{jw})$ can only be evaluated at a prescribed set of discrete frequencies. Moreover, only a class of the DTFT that is expressed as a rational function in e^{-jw} in the form

$$X(e^{jw}) = \frac{p_0 + p_1 e^{-jw} + \cdots + p_M e^{-jwM}}{d_0 + d_1 e^{-jw} + \cdots + d_N e^{-jwN}} \tag{3.31}$$

can be evaluated.

Project 1: DTFT computation

Program P3_1: used to evaluate and plot the DTFT of the form of Eq. (3.31)

```
% Program P3_1
% Evaluation of the DTFT
```

```
clf;
% Compute the frequency samples of the DTFT
w = -4*pi:8*pi/511:4*pi;
num = [2 1];den = [1 -0.6];
h = freqz(num, den, w);
% Plot the DTFT
subplot(2,1,1)
plot(w/pi,real(h));grid
title('Real part of H(e^{j\omega})')
xlabel('\omega /\pi');
ylabel('Amplitude');
subplot(2,1,2)
plot(w/pi,imag(h));grid
title('Imaginary part of H(e^{j\omega})')
xlabel('\omega /\pi');
ylabel('Amplitude');
pause
subplot(2,1,1)
plot(w/pi,abs(h));grid
title('Magnitude Spectrum |H(e^{j\omega})|')
xlabel('\omega /\pi');
ylabel('Amplitude');
subplot(2,1,2)
plot(w/pi,angle(h));grid
title('Phase Spectrum arg[H(e^{j\omega})]')
xlabel('\omega /\pi');
ylabel('Phase in radians');
```

Questions:

Q1 What is the expression of the DTFT being evaluated in Progrem P3_1? What is the function of the MATLAB command pause?

Q2 Run Progrem P3_1 and compute the real and imaginary parts of the DTFT, and the magnitude and phase spectra. Is the DTFT a periodic function of *w*? If it is, what is the period? Explain the type of symmetries exhibited by the four plots.

Q3 Modify Progrem P3_1 to evaluate in the range $0 \le w \le \pi$ the following DTFT:

$$U\left(e^{jw}\right)=\frac{0.7-0.5e^{-jw}+0.3e^{-j2w}+e^{-j3w}}{1+0.3e^{-jw}-0.5e^{-j2w}+0.7e^{-j3w}}$$

And repeat Question Q2. Comment on your results. Can you explain the jump in the phase spectrum? The jump can be removed using the MATLB command unwrap. Evaluate the phase spectrum with the jump removed.

Q4 Modify Progrem P3_1 to evaluate the DTFT of the following finite-length sequence:

$$g[n] = [1\ 3\ 5\ 7\ 9\ 11\ 13\ 15\ 17]$$

And repeat Question Q2. Comment on your results. Can you explain the jump in the phase spectrum?

Project 2: DTFT properties

Program P3_2: can be used to verify the time-shifting property of the DTFT

```
% Program P3_2
% Time-Shifting Properties of DTFT
clf;
w = -pi:2*pi/255:pi; wo = 0.4*pi; D = 10;
num = [1 2 3 4 5 6 7 8 9];
h1 = freqz(num, 1, w);
h2 = freqz([zeros(1,D) num], 1, w);
subplot(2,2,1)
plot(w/pi,abs(h1));grid
title('Magnitude Spectrum of Original Sequence')
subplot(2,2,2)
plot(w/pi,abs(h2));grid
title('Magnitude Spectrum of Time-Shifted Sequence')
subplot(2,2,3)
plot(w/pi,angle(h1));grid
title('Phase Spectrum of Original Sequence')
subplot(2,2,4)
plot(w/pi,angle(h2));grid
title('Phase Spectrum of Time-Shifted Sequence')
```

Question:

Q5 Modify Program P3_2 by adding appropriate comment statements and program statements for labeling the two axes of each plot being generated by the program. Which parameter controls the amount of time shift:

Program P3_3: can be used to verify the frequency-shifting property of the DTFT

```
% Program P3_3
% Frequency-Shifting Properties of DTFT
clf;
w = -pi:2*pi/255:pi; wo = 0.4*pi;
```

```
num1 = [1 3 5 7 9 11 13 15 17];
L = length(num1);
h1 = freqz(num1, 1, w);
n = 0:L-1;
num2 = exp(wo*i*n).*num1;
h2 = freqz(num2, 1, w);
subplot(2,2,1)
plot(w/pi,abs(h1));grid
title('Magnitude Spectrum of Original Sequence')
subplot(2,2,2)
plot(w/pi,abs(h2));grid
title('Magnitude Spectrum of Frequency-Shifted Sequence')
subplot(2,2,3)
plot(w/pi,angle(h1));grid
title('Phase Spectrum of Original Sequence')
subplot(2,2,4)
plot(w/pi,angle(h2));grid
title('Phase Spectrum of Frequency-Shifted Sequence')
```

Questions:

Q6 Modify Program P3_3 by adding appropriate comment statements and program statements for labeling the two axes of each plot being generated by the program. Which parameter controls the amount of frequency-shift.

Program P3_4: can be used to verify the convolution property of the DTFT

```
% Program P3_4
% Convolution Property of DTFT
clf;
w = -pi:2*pi/255:pi;
x1 = [1 3 5 7 9 11 13 15 17];
x2 = [1 -2 3 -2 1];
y = conv(x1,x2);
h1 = freqz(x1, 1, w);
h2 = freqz(x2, 1, w);
hp = h1.*h2;
h3 = freqz(y,1,w);
subplot(2,2,1)
plot(w/pi,abs(hp));grid
title('Product of Magnitude Spectra')
```

```
subplot(2,2,2)
plot(w/pi,abs(h3));grid
title('Magnitude Spectrum of Convolved Sequence')
subplot(2,2,3)
plot(w/pi,angle(hp));grid
title('Sum of Phase Spectra')
subplot(2,2,4)
plot(w/pi,angle(h3));grid
title('Phase Spectrum of Convolved Sequence')
```

Question:

Q7 Modify Program P3_4 by adding appropriate comment statements and program statements for labeling the two axes of each plot being generated by the program.

Program P3_5: can be used to verify the modulation property of the DTFT

```
% Program P3_5
% Modulation Property of DTFT
clf;
w = -pi:2*pi/255:pi;
x1 = [1 3 5 7 9 11 13 15 17];
x2 = [1 -1 1 -1 1 -1 1 -1 1];
y = x1.*x2;
h1 = freqz(x1, 1, w);
h2 = freqz(x2, 1, w);
h3 = freqz(y,1,w);
subplot(3,1,1)
plot(w/pi,abs(h1));grid
title('Magnitude Spectrum of First Sequence')
subplot(3,1,2)
plot(w/pi,abs(h2));grid
title('Magnitude Spectrum of Second Sequence')
subplot(3,1,3)
plot(w/pi,abs(h3));grid
title('Magnitude Spectrum of Product Sequence')
```

Question:

Q8 Modify Program P3_5 by adding appropriate comment statements and program statements for labeling the two axes of each plot being generated by the program.

Program P3_6: can be used to verify the time-reversal property of the DTFT

```
% Program P3_6
% Time Reversal Property of DTFT
clf;
w = -pi:2*pi/255:pi;
num = [1 2 3 4];
L = length(num)-1;
h1 = freqz(num, 1, w);
h2 = freqz(fliplr(num), 1, w);
h3 = exp(w*L*i).*h2;
subplot(2,2,1)
plot(w/pi,abs(h1));grid
title('Magnitude Spectrum of Original Sequence')
subplot(2,2,2)
plot(w/pi,abs(h3));grid
title('Magnitude Spectrum of Time-Reversed Sequence')
subplot(2,2,3)
plot(w/pi,angle(h1));grid
title('Phase Spectrum of Original Sequence')
subplot(2,2,4)
plot(w/pi,angle(h3));grid
title('Phase Spectrum of Time-Reversed Sequence')
```

Question:

Q9 Modify Program P3_6 by adding appropriate comment statements and program statements for labeling the two axes of each plot being generated by the program. Explain how the program implements the time-reversal operation.

3.3 Discrete Fourier Transform

The discrete Fourier transform (DFT) $X[k]$ of a finite-length $x[n]$ can be easily computed in MATLAB using the function fft. There are two versions of this function, fft(x) computes the DFT $X[k]$ of the sequence $x[n]$ where the length of $X[k]$ is the same as that $x[n]$. fft(x,L) computes the L-point of a sequence $x[n]$ of length N where $L \geq N$. If $L > N$, $x[n]$ is zero-padded with $L-N$ trailing zero-valued samples before the DFT is computed. The inverse discrete Fourier transform (IDFT) $x[n]$ of a DFT sequence $X[k]$ can be computed using the function ifft, which also has two versions.

Project 3: DFT and IDFT computations

Question:

Q10 Write a MATLAB program to compute and plot the L-point DFT $X[k]$ of a sequence $x[n]$ of length N with $L \geq N$ and then to compute and plot the L-point DFT of $X[k]$. Run the program for sequences of different lengths N and for different values of the DFT length L. Comment on your results.

Project 4: DFT properties

Two important concepts used in the application of the DFT are the circular-shift of a sequence and the circular convolution of two sequences of the same length. As these operations are needed in verifying certain properties of the DFT, we implement them as MATLAB functions circshift and circonv.

```
function y = circshift(x,M)
% Develops a sequence y obtained by
% circularly shifting a finite-length
% sequence x by M samples
if abs(M) > length(x)
M = rem(M,length(x));
end
if M < 0
M = M + length(x);
end
y = [x(M+1:length(x)) x(1:M)];

function y = circonv(x1,x2)
L1 = length(x1); L2 = length(x2);
if L1 ~= L2, error('Sequences of unequal lengths'), end
y = zeros(1,L1);
x2tr = [x2(1) x2(L2:-1:2)];
for k = 1:L1
sh = circshift(x2tr,1-k);
h = x1.*sh;
y(k) = sum(h);
end
```

Questions:

Q11 What is the purpose of the command rem in the function circshift?

Q12 Explain how the function circshift implements the circular time-shifting operation.

Q13 What is the purpose of the operator ~= in the function circonv?

Q14 Explain how the function circonv implements the circular convolution operation.

Program P3_7: can be used to illustrate the concept of circular shift of a finite-length sequence.

```
% Program P3_7
% Illustration of Circular Shift of a Sequence
clf;
M = 6;
a = [0 1 2 3 4 5 6 7 8 9];
b = circshift(a,M);
L = length(a)-1;
n = 0:L;
subplot(2,1,1);
stem(n,a);axis([0,L,min(a),max(a)]);
title('Original Sequence');
subplot(2,1,2);
stem(n,b);axis([0,L,min(a),max(a)]);
title(['Sequence Obtained by Circularly Shifting by ',num2str(M),' Samples']);
```

Question:

Q15 Modify Program P3_7 by adding appropriate comment statements and program statements for labeling each plot being generated by the program. Which parameter determines the amount of time-shifting? What happens if the amount of time-shift is greater than the sequence length?

Program P3_8: can be used to illustrate the circular time-shifting operation.

```
% Program P3_8
% Circular Time-Shifting Property of DFT
clf;
x = [0 2 4 6 8 10 12 14 16];
N = length(x)-1; n = 0:N;
y = circshift(x,5);
XF = fft(x);
YF = fft(y);
subplot(2,2,1)
stem(n,abs(XF));grid
title('Magnitude of DFT of Original Sequence');
subplot(2,2,2)
stem(n,abs(YF));grid
title('Magnitude of DFT of Circularly Shifted Sequence');
subplot(2,2,3)
```

```
stem(n,angle(XF));grid
title('Phase of DFT of Original Sequence');
subplot(2,2,4)
stem(n,angle(YF));grid
title('Phase of DFT of Circularly Shifted Sequence');
```

Question:

Q16 Modify Program P3_8 by adding appropriate comment statements and program statements for labeling each plot being generated by the program. What is the amount of time-shift?

Program P3_9: can be used to illustrate the circular convolution property of the DFT.

```
% Program P3_9
% Circular Convolution Property of DFT
g1 = [1 2 3 4 5 6]; g2 = [1 -2 3 3 -2 1];
ycir = circonv(g1,g2);
disp('Result of circular convolution = ');disp(ycir)
G1 = fft(g1); G2 = fft(g2);
yc = real(ifft(G1.*G2));
disp('Result of IDFT of the DFT products = ');disp(yc)
```

Question:

Q17 Run Program P3_9 and verify the circular convolution property of the DFT.

Program P3_10: can be used to illustrate the relation between circular and linear convolutions.

```
% Program P3_10
% Linear Convolution via Circular Convolution
g1 = [1 2 3 4 5];g2 = [2 2 0 1 1];
g1e = [g1 zeros(1,length(g2)-1)];
g2e = [g2 zeros(1,length(g1)-1)];
ylin = circonv(g1e,g2e);
disp('Linear convolution via circular convolution = ');disp(ylin);
y = conv(g1, g2);
disp('Direct linear convolution = ');disp(y)
```

Question:

Q18 Run Program P3_10 and verify that linear convolution can be obtained via circular convolution.

Program P3_11: can be used to illustrate the relation between the DFTs of the periodic even and the periodic odd parts of a real sequence, and its DFT.

```
% Program P3_11
% Relations between the DFTs of the Periodic Even
% and Odd Parts of a Real Sequence
x = [1 2 4 2 6 32 6 4 2 zeros(1,247)];
x1 = [x(1) x(256:-1:2)];
xe = 0.5 *(x + x1);
XF = fft(x);
XEF = fft(xe);
clf;
k = 0:255;
subplot(2,2,1);
plot(k/128,real(XF)); grid;
ylabel('Amplitude');
title('Re(DFT\{x[n]\})');
subplot(2,2,2);
plot(k/128,imag(XF)); grid;
ylabel('Amplitude');
title('Im(DFT\{x[n]\})');
subplot(2,2,3);
plot(k/128,real(XEF)); grid;
xlabel('Time index n');ylabel('Amplitude');
title('Re(DFT\{x_{e}[n]\})');
subplot(2,2,4);
plot(k/128,imag(XEF)); grid;
xlabel('Time index n');ylabel('Amplitude');
title('Im(DFT\{x_{e}[n]\})');
```

Questions:

Q19 What is the relation between the sequences $x1[n]$ and $x[n]$?

Q20 Run Program P3_11. The imaginary part of XEF should be zero as the DFT of the periodic even part is simply the real part of XEF of the original sequence. Can you verify that? How can you explain the simulation result?

Q21 Modify the program to verify the relation between the DFT of the periodic odd part and the imaginary part of XEF.

Program P3_12: can be used to verify Parseval's relation.

```
% Program P3_12
% Parseval's Relation
```

```
x = [(1:128) (128:-1:1)];
XF = fft(x);
a = sum(x.*x)
b = round(sum(abs(XF).^2)/256);
```

Questions:

Q22 Run Program P3_12. Do you get the same values for a and b?